Early evolution of transversally thermalized partons ⋆

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Abstract

The idea that the parton system created in relativistic heavy-ion collisions (i) emerges in a state with transverse momenta close to thermodynamic equilibrium and (ii) its evolution at early times is dominated by the 2-dimensional (transverse) hydrodynamics of the ideal fluid is investigated. It is argued that this mechanism may help to solve the problem of early equilibration.

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1. It is now commonly accepted that evolution of the partonic system created in heavy-ion collisions at RHIC energies is best described by hydrodynamics of an almost ideal fluid [1]. In particular, particle transverse-momentum spectra and asymmetry of the transverse flow, as represented by the parameter v_2 [2], are reasonably well reproduced by the hydrodynamic approach.

At the same time it is also realized that the hydrodynamic picture encounters a rather serious challenge: it requires very early thermalization of the system. This follows from the fact that the asymmetry of the transverse flow is produced most effectively at the very early stage of the evolution (when

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the pressure gradients are largest). To obtain the asymmetry parameter v_2 consistent with data it is necessary to start the hydrodynamic evolution at the time below 1 fm after the collision takes place. But application of hydrodynamics demands, as the necessary condition, the local equilibration of the system. Such fast equilibration is not easy to achieve with elastic perturbative cross-sections. This puzzle of early thermalization was also widely discussed and several exotic mechanisms were proposed for its solution [3,4]. It seems fair to say, however, that no one was yet accepted as fully satisfactory.

In the present paper we explore the possibility that, at its early stages, the hydrodynamic evolution applies only to transverse degrees of freedom of the partonic system created in high-energy collisions. The main reason of interest in such a study is the observation that the early equilibration, if at all possible, is particularly difficult to achieve in longitudinal direction. This is because an elastic collision does not change significantly the direction of the colliding partons and thus it requires very many interactions to produce a locally isotropic distribution from the initially strongly anisotropic one [1].

At the same time one may argue that the equilibration of the partonic spectrum in transverse direction can be obtained much easier. Indeed, it is well known that the transverse momentum spectra observed in nucleon-nucleon collisions are well described by the Boltzmann distribution [5,6]. As these spectra reflect - at least to some extent - the distribution of produced partons [7,8,9], it is clear that the partonic system produced in hadronic collisions emerges already in a state close enough to equilibrium *in the transverse direction* so that no much more action is needed to achieve the goal, see e.g. [10,11,12].

At this point it is important to emphasize that such a description may be adequate only during a certain time after the collision. Indeed, as the time goes on, the interpartonic interactions will tend to equilibrate the system also in longitudinal direction and thus the standard, 3-dimensional hydrodynamics will probably take over.

To study the possibility of such purely transverse hydrodynamic evolution and to have also a connection with physics of the relativistic heavy-ion collisions we discuss a simple model implementing these ideas. The parton distribution is constructed as a superposition of many clusters whose longitudinal motion satisfies the Bjorken in-out condition $\eta = y$ where

$$\eta = \frac{1}{2} \log \frac{t+z}{t-z} \tag{1}$$

is the spatial rapidity of the cluster and y is its rapidity. Transverse momenta

[1] We thank B. Mueller for an interesting conversation about this point.

in each cluster follow the equilibrium distribution in the local rest frame of the fluid.

This is realized by the following Ansatz for the parton distribution function

$$F(x,p) = f_{\|} \; g(\tau, \eta; \vec{x}_{\perp}, \vec{p}_{\perp}), lans \tag{2}$$

where $f_{\|}$ describes the longitudinal motion of clusters,

$$f_{\|} = n_0 \delta(p_z t - Ez) = n_0 \frac{\delta(y-\eta)}{\tau m_{\perp}}, lfpar \tag{3}$$

with τ being the longitudinal proper time, $\tau^2 = t^2 - z^2$, n_0 is the normalization factor giving the density of clusters per unit of rapidity, and g represents the distribution of transversally equilibrated partons. In our study we neglect parton masses and take for g the Boltzmann distribution

$$g = \exp\left(-\frac{p_\mu U^\mu}{T}\right), ldisg \tag{4}$$

where U^μ is the four velocity of the fluid,

$$U^\mu = (u_0 \cosh y, u_x, u_y, u_0 \sinh y) = (u_0 \cosh \eta, u_x, u_y, u_0 \sinh \eta), \tag{5}$$

with u^μ being the 4-velocity of the fluid in the frame where the longitudinal momentum vanishes

$$u^\mu = (u_0, u_x, u_y, 0) = (u_0, \vec{u}_{\perp}, 0), \quad u_0^2 - \vec{u}_{\perp}^{\,2} = 1.lumu \tag{6}$$

Using these ideas, the hydrodynamic equations were derived from the Ansatz (**??**) – (**??**) and the transverse expansion of the fluid was studied numerically for various initial and final conditions.

Starting from the initial profile determined by the density of the wounded nucleons inside the colliding nuclei, we have found that

(i) With the proper choice of the initial temperature T_i it is possible to obtain parton spectra that are consistent with the experimentally observed pion transverse-momentum distributions.

(ii) The calculated elliptic flow parameter v_2 agrees with data, provided the final temperature T_f (i.e. the temperature of transition from 2-dimensional (2D) to 3-dimensional (3D) regime) is taken substantially higher than the

expected freeze-out temperature. This gives a rather short time available for the 2D evolution.

(iii) This short evolution time implies that the transverse size of the system is relatively small, smaller than that obtained from the HBT measurements, thus consistently leaving space for further, 3D expansion of the system.

We thus conclude that the 2D hydrodynamics may indeed be a reasonable description of the partonic system during the first few fermis after the collision and it gives a good starting point for a realistic treatment. If confirmed by more detailed studies, this would imply serious changes in the present understanding of the quark-gluon plasma evolution.

These conclusions differ substantially from those reached in a study of a similar problem by Heinz and Wong [13]. Using a different implementation of the ideas advocated in the present paper, they found that it is not possible to obtain a satisfactory description of the elliptic flow and concluded that the partonic system must be necessarily in a 3D equilibrium from the very beginning of the evolution. Our example shows that this is not necessarily the case. In the last section we discuss the possible reasons for this discrepancy.

In the next section we discuss the general form of the energy-momentum tensor following from the Ansatz (**??**) – (**??**) and the corresponding hydrodynamical equations. The numerical analysis of the evolution of the system are presented in Section 3. The last section summarizes our conclusions.

2. To obtain the hydrodynamic equations we first derive, using the Ansatz (**??**) – (**??**), the general form of the particle density and entropy density 4-vectors and of the energy-momentum tensor. Somewhat lenghty but straightforward calculations give (with $\hbar = c = 1$)

$$N^\mu = n_0\, \nu_g \int dy \frac{d^2p_\perp}{(2\pi)^2} p^\mu f_\parallel\, g = n_0\, \nu_g \frac{T^2}{2\pi\tau} U^\mu, ldensn \tag{7}$$

$$T^{\mu\nu} = n_0\, \nu_g \int dy \frac{d^2p_\perp}{(2\pi)^2} p^\mu p^\nu f_\parallel\, g = n_0\, \nu_g \frac{T^3}{2\pi\tau} \left[3U^\mu U^\nu - g^{\mu\nu} - V^\mu V^\nu\right], ldens \tag{8}$$

where ν_g is the number of internal degrees of freedom ($\nu_g = 16$ for gluons) and

$$V^\mu = (\sinh\eta, 0, 0, \cosh\eta) \tag{9}$$

is the 4-vector defining the longitudinal (z) direction. It is also interesting to evaluate the entropy flow, giving

$$S^\mu = -n_0\,\nu_g \int dy \frac{d^2p_\perp}{(2\pi)^2} p^\mu f_\| \, g(\log g - 1) = n_0\,\nu_g \frac{3T^2}{2\pi\tau} U^\mu .ldenss \qquad (10)$$

From these results we also read that the energy, entropy and particle number densities in the local rest frame are

$$\frac{dE}{d^2x_\perp dz} = n_0\,\nu_g \frac{T^3}{\pi\tau}\;;\quad \frac{dN}{d^2x_\perp dz} = \frac{1}{3}\frac{dS}{d^2x_\perp dz} = n_0\,\nu_g \frac{T^2}{2\pi\tau}. \qquad (11)$$

where the equation of state $dE/(d^2x_\perp dz) = 2P$ was used. Note that dependence of these densities on temperature is weaker than that known from 3D thermodynamics.

Using $dz = \tau d\eta$ we thus obtain the densities per unit of spatial rapidity

$$\epsilon_2 \equiv \frac{dE}{d^2x_\perp d\eta} = n_0\,\nu_g \frac{T^3}{\pi}\;;\quad n_2 \equiv \frac{dN}{d^2x_\perp d\eta} = n_0\,\nu_g \frac{T^2}{2\pi} = \frac{1}{3}\frac{dS}{d^2x_\perp d\eta} \equiv \frac{1}{3}s_2, \qquad (12)$$

which are *identical* to the densities proper for 2D thermodynamics.

The hydrodynamic equations are obtained from the energy-momentum conservation laws $\partial_\mu T^{\mu\nu} = 0$ which also imply the conservation of entropy $\partial_\mu S^\mu = 0$. Entropy conservation implies

$$\partial_\tau [u_0 T^2] + \vec{\nabla}_\perp \cdot [\vec{u}_\perp T^2] = 0, leqs \qquad (13)$$

while the momentum conservation provides two other equations

$$\partial_\tau [\vec{u}_\perp T^3 u_0] + [T^3 \vec{u}_\perp](\vec{\nabla}_\perp \cdot \vec{u}_\perp) + (\vec{u}_\perp \cdot \vec{\nabla}_\perp)[T^3 \vec{u}_\perp] + \vec{\nabla}_\perp T^3/3 = 0.leqt \qquad (14)$$

These are three equations for the three unknowns: the temperature T, and two independent components of the 4-velocity u_x and u_y. Their characteristic feature is that they do not depend explicitly on the space-time variables (τ, x, y). Thus, they do not select any special point in space time. We have solved these equations numerically with the method described in [14] which is a direct extension of that proposed in [15]. The details of the procedure are published in [16].

3. To study the evolution of the system following from (**??**) and (**??**) we need to specify the initial conditions, adequate for the physical situation encountered in $Au-Au$ collisions at RHIC energy. To this end we assume that the profile (in

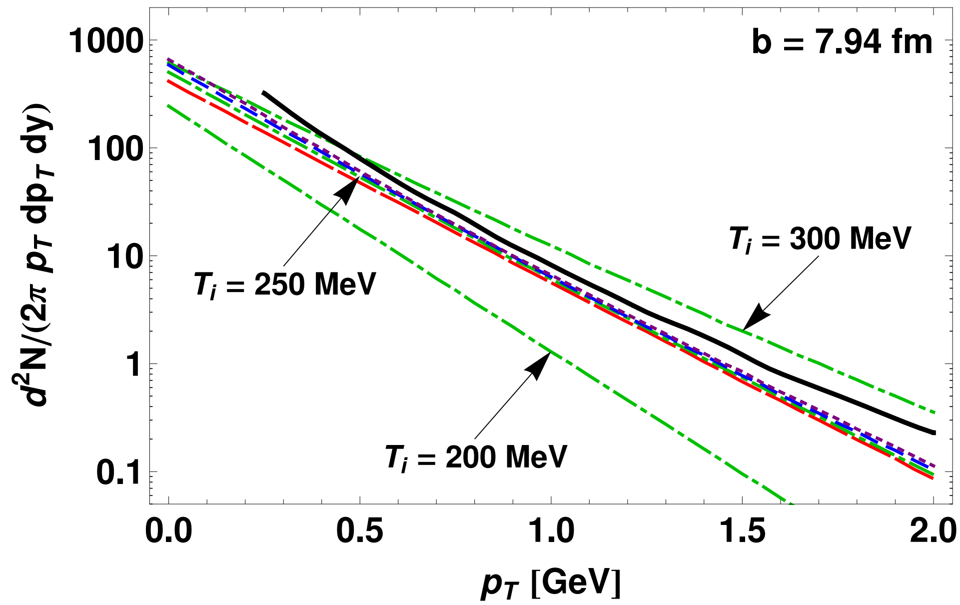

Fig. 1. Transverse-momentum spectra of positive pions measured by the PHENIX Collaboration in the centrality class 30-40% (solid line) [18] and the model spectra of gluons for various choices of T_i and T_f. The lowest dash dotted curve corresponds to $T_i = 200$ MeV and $T_f = 180$ MeV. The highest dash dotted curve was obtained for $T_i = 300$ MeV and $T_f = 180$ MeV. The four almost parallel lines represent our results for $T_i = 250$ MeV and for four final values of the temperature: $T_f = 200$ (long dashed line), 180 (dash dotted line), 160 (dashed line) and 140 (dotted line) MeV. Note that the original π^+ experimental spectra collected at $\sqrt{s_{NN}} = 200$ GeV have been multiplied by a factor of 3 to account for the total hadron multiplicity.

transverse coordinates) of the initial energy density ϵ_2 is given by the density of participants. This density is determined, for a given centrality, from the Glauber formulae. The following discussion is given for centrality $20-40\%$, corresponding to impact parameter $b \approx 7.9$ fm. The initial temperature T_i at the origin, $\vec{x}_\perp = 0$, is taken as a free parameter.

The parton spectra are evaluated using the Cooper-Frye prescription [17]

$$\frac{dN}{d^2p_\perp dy} = \frac{n_0\,\nu_g}{(2\pi)^2} \int d\Sigma_\mu(x) p^\mu\, f_\| \,g, lcp \tag{15}$$

where Σ is the surface at which the 2D evolution comes to the end. In our exercise we have taken Σ to be the surface of constant temperature.

The parton transverse-momentum spectrum evaluated from the model depends substantially on the initial temperature of the system but is practically insensitive to the final temperature (this observation was already made in [13]). In Fig. 1, the spectrum obtained for various values of initial temperature T_i is compared with measured spectrum of pions [18]. One sees that the

correct slope [2] is obtained for $T_i \approx 250$ MeV. This initial temperature is used in all subsequent calculations. The normalization, which is arbitrary in our model, was calculated with $n_0 = 1$.

At the fixed initial temperature, results for elliptic flow are sensitive to the value of the final temperature T_f. This is shown in Fig. 2 where v_2 calculated from the model is plotted versus $p_\perp$ for various final temperatures. One sees that, as expected, v_2 grows with decreasing T_f (at a fixed T_i, smaller T_f means longer time of evolution and thus more time to develop the flow). As already observed in [13], the absence of longitudinal pressure implies that the obtained values of v_2 are higher than those expected from 3D evolution (with the same initial conditions). To bring v_2 close to the observed one [19], a fairly high $T_f \approx 180$ MeV is needed. This agrees with the point of view formulated in the first section: The final temperature should not be interpreted as the "freeze-out" temperature but rather as the temperature when the 2D character of the equilibrium changes into the 3D one. This may happen well before the freeze-out and hadronization. So the high value of T_f is not surprising.

The fact the v_2 grows with time indicates that our approach can be adequate only at the initial stage of the evolution of the system. After this initial stage other effects such as the early transition to 3D hydro with a reduced sound velocity or with viscous effects, hadronization, and switching to transport description will reduce the growth of v_2 shown in Fig. 2.

Having fixed the initial and final temperatures, we can also estimate the lifetime of the 2D evolution. It turns out to be of about 4 fermi (for the $20-40\%$ centrality class).

4. In conclusion, we have investigated the consequences of the hypothesis that the partonic system produced in a collision of two heavy ions is created in a state close to thermodynamical equilibrium *in transverse direction*, while its longitudinal structure is characterized by a freely-streaming collection of clusters. Two observables were studied using the Cooper-Frye formula at the surface of constant temperature with the initial energy density profile determined from the density of the number of participants at a given centrality.

(a) The slope of the transverse momentum spectrum was adjusted to the one measured for the pion spectra [18] giving the initial temperature at the center of the system $T_i = 250$ MeV.

(b) The calculated elliptic flow parameter v_2 gives the final temperature (at

[2] At this stage the experimental shape of the spectrum is not correctly reproduced since we do not include the effects such as, e.g., resonance decays and hard scattering.

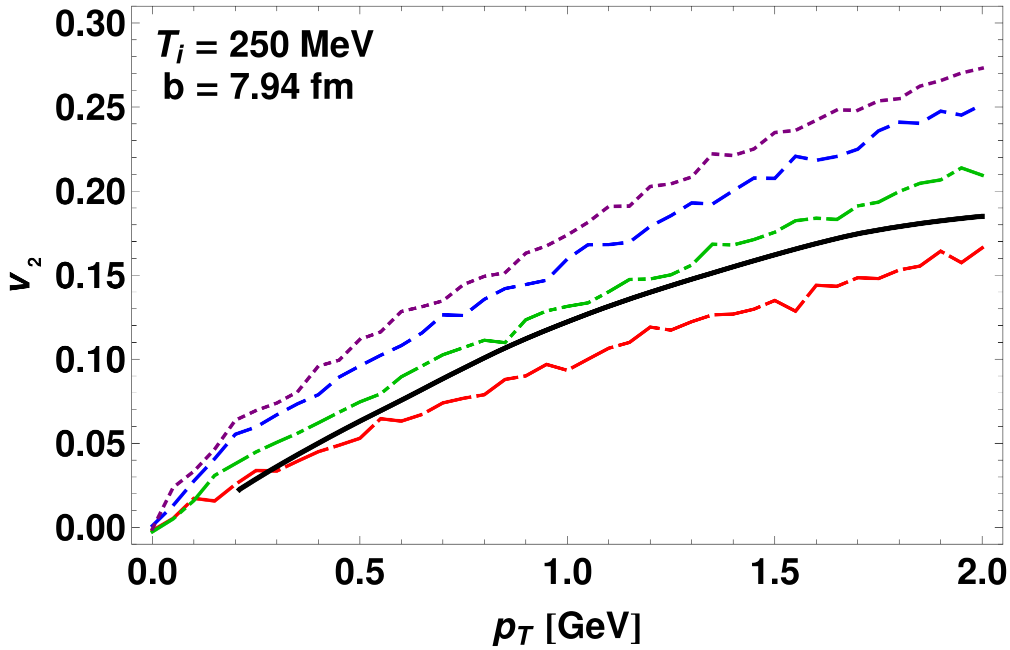

Fig. 2. The elliptic flow coefficient v_2 as the function of the transverse momentum. The PHENIX experimental results for pions and kaons in the centrality class 20-40% and for the collision energy $\sqrt{s_{NN}} = 200$ GeV [19] are compared to the model calculations with $T_i = 250$ MeV and for four final values of the temperature $T_f = 200$, 180, 160 and 140 MeV. The curves are denoted in the same way as in Fig. 1. The best agreement is obtained for $T_f \approx 180$ MeV.

which the transition from 2D to 3D regime takes place) $T_f = 180$ MeV, substantially higher than the expected hadronization temperature. The time needed for the 2D evolution was estimated to be much shorter than that normally needed to achieve the freeze-out. This confirms the point of view that the equilibrium changes from 2D to 3D well before the freeze-out and hadronization.

We thus conclude that the initial period of 2D transverse equilibration and hydrodynamic evolution of the parton system is helping to solve the problem of early equilibration, most pertinent difficulty of the present application of hydrodynamics to physics of heavy-ion collisions. It would be certainly interesting to investigate this possibility in more detail.

Several comments are in order.

(i) Our conclusions differ from those obtained in [13]. As far as we can see, apart from certain technical details, there are essentially three reasons for this discrepancy.

First, the density distributions implementing the assumption of free-streaming in the longitudinal direction differ by a factor depending on the transverse mass. This leads to rather serious consequences. Our ansatz (2) - (4) implies

that at fixed rapidity the evolution of system obeys the rules of truly 2D thermodynamics of an *ideal fluid*, whereas in [13] the temperature dependence of the energy and number densities follows that of 3D thermodynamics and the resulting hydrodynamic equations should be interpreted as an effective description of the *viscous hydrodynamics*. Our energy-momentum tensor is more symmetric and, consequently, the hydrodynamic equations differ from those used in [13]. We stress that our approach is thermodynamically consistent and the conservation laws for the energy and momentum, $\partial_\mu T^{\mu\nu} = 0$, lead to the entropy conservation, $\partial_\mu S^\mu = 0$, in the way very much similar as in the standard relativistic hydrodynamics.

Second, in Ref. [13] a new timescale parameter τ_0 is introduced. This parameter – absent in our formulation – plays an important role in the argument used in [13] to reject a two-dimensional evolution. The large values of τ_0, found in the fitting procedure by Heinz and Wong, are in their opinion inconsistent with the very idea of transverse thermalization because such thermalization may take place only at the beginning of the evolution of the system, when the considered times τ are small (with $\tau_0 < \tau$). The replacement $\tau_0 \to n_0/m_\perp$ (which formally transforms their model into our approach) solves this problem if n_0 is interpreted as the density of 2D clusters in rapidity.

Third, the authors of [13] confront their results for the elliptic flow with the earlier 3D hydrodynamic calculations, whereas we fix our parameters by comparison with the present data. Our results shown in Fig. 2 indicate that we can reproduce the experimental values of v_2 already at high values of the temperature where most of the effect is created. The 3D hydro results shown in Fig. 7 of [13] are above the data, hence they cannot be used as the reference point for rejecting the concept of transverse thermalization.

(ii) In our calculations we have assumed a sharp transition between the 2D and a possible 3D evolution. This simplification, necessary to obtain a reasonably tractable problem, can be removed in future, more sophisticated analyses. This raises interesting questions about the nature of this transition and about the final fate of the clusters.

(iii) It should be clear that our investigation represents only the first step towards a fully realistic description. Many details, as e.g. the mechanism of 2D $\to$ 3D transition, the relative distribution of clusters, their internal (longitudinal) structure, the rapidity dependence of the system, are left for future work. Nevertheless, we feel that we produced some compelling arguments for the existence of a substantial period of purely transverse thermodynamic equilibrium and hydrodynamic evolution of the partonic system (in the form of the ideal fluid) at the very beginning of the collision process.

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References

[1] For a review see, e.g., E.Shuryak, Nucl. Phys. A750 (2005) 64.

[2] J. Y. Ollitrault, Phys. Rev. D46 (1992) 229.

[3] St. Mrówczyński, Acta Phys. Pol. B37 (2006) 427.

[4] M. Strickland, talk at Quark Matter 2006, hep-ph/0701238

[5] F. Becattini, Z. Phys. C69 (1996) 485.

[6] F. Becattini and U. Heinz, Z. Phys. C76 (1997) 269, hep-ph/9702274.

[7] A. Bialas, M. Bleszynski, and W. Czyz, Nucl. Phys. B111 (1976) 461.

[8] A. Bialas and W. Czyz, Acta. Phys. Pol. B36 (2005) 905.

[9] A. Bialas and A. Bzdak, Phys. Lett. B649 (2007) 263.

[10] A. Bialas, Phys. Lett. B466 (1999) 301, hep-ph/9909417.

[11] W. Florkowski, Acta Phys. Pol. B35 (2004) 799, nucl-th/0309049.

[12] S. Steinke and J. Rafelski, J. Phys. G32 (2006) S455, nucl-th/0607066.

[13] U. Heinz and S.M.H. Wong, Phys. Rev. C66 (2002) 014907, and U. Heinz, private communication.

[14] M. Chojnacki, W. Florkowski, Phys. Rev. C74 (2006) 034905, nucl-th/0603065.

[15] G. Baym, B. L. Friman, J. P. Blaizot, M. Soyeur, W. Czyz, Nucl. Phys. A407 (1983) 541.

[16] M. Chojnacki and W. Florkowski, Acta Phys. Pol. B39 (2008) 721, arXiv:0710.5871 [nucl-th].

[17] F. Cooper and G. Frye, Phys. Rev. D10 (1974) 186.

[18] S. Adler et al., PHENIX Collaboration, Phys. Rev. C69 (2004) 034909, nucl-ex/0307022

[19] S. Adler et al., PHENIX Collaboration, Phys. Rev. Lett. 91 (2003) 182301, nucl-ex/0305013.